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ABSTRACT

The robustness with respect to Type I error and the power of a proposed test statistic in testing the conjoint hypotheses of mean and variability equality were examined in this simulation study. The conjoint test utilizes the maximum p-value from separate tests of equality of means and equality of variability as its p-value to control the Type I error rates. The number of groups (2, 4, 6, 8, and 10), number of subjects per group (10, 20, 40 and 80), and distribution shapes (4) were manipulated. Data with equal means and equal variances, equal means and unequal variances, and unequal means and equal variances were simulated to obtain the Type I error rates; whereas data with unequal means and unequal variances were simulated to obtain the empirical power. Results showed that the conjoint test yielded low Type I error under all three conditions across the manipulated variables. The conjoint test provided "reasonable" power when the sample size per group was large. An appendix presents a sample computer program for the analysis. (Contains five references and seven tables.) (Author/SLD)

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ED 398 272

RUNNING HEAD: CONJOINT TEST IN ONE-WAY LAYOUT

A Conjoint Test for Testing the Equality of Mean and Variability in a One-way ANOVA Layout

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Paper presented at the annual meeting of the American Educational Research Association, April 8-12, 1996, New York.

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Abstract

The robustness with respect to the Type I error and the power of a proposed test statistic in testing the conjoint hypotheses of mean and variability equality were examined in this simulation study. The conjoint test utilizes the maximum p-value from separate tests of equality of means and equality of variability as its p-value to control the Type I error rates. The number of groups, number of subjects per group, and distribution shapes were manipulated. Data with equal means and equal variances, equal means and unequal variances, and unequal means and equal variances were simulated to obtain the Type I error rates; whereas data with unequal means and unequal variances were simulated to obtain the empirical power. Results showed that the conjoint test yielded low Type I error under all three conditions across the manipulated variables. The conjoint test provided "reasonable" power when the sample size per group was "large."

A Conjoint Test for Testing the Equality of Mean and Variability in a One-way ANOVA Layout

To deal with the situation where the experimental treatments are expected to increase the experimental group variability relative to that of the control group, Brownie, Boos, and Hughes-Oliver (1990) proposed a modified F statistic (Fmod) to simultaneously test the null hypotheses that all the group means and variances are equal against the alternative hypotheses that not all the group means are equal and the variability of the experimental groups are greater than that of the control.

$$Ho: \mu_c = \mu_{exp} \quad \sigma^2_c = \sigma^2_{exp}$$

vs.

$$Ha: \mu_c \neq \mu_{exp} \quad p\sigma^2_c = \sigma^2_{exp}$$

where c denotes the control group, exp denotes the experimental group, and $1 \leq p \leq \infty$.

The test statistic Brownie, et al. (1990) proposed has the same basic format as an F test in ANOVA. It differs in that it uses the variance of the control group in place of the usual within group mean square, and the referent distribution is F with k-1 and n_c-1 degrees of freedom, where k is the number of groups and n_c is the number of subjects in the control group.

$$Fmod = [\Sigma n_i(x_i - x..)^2/(k-1)]/S^2_c \rightarrow F(k-1, n_c-1)$$

They claimed that this Fmod test would be more powerful and robust with respect to Type I error than the regular F test when the variability of the treatment groups are expected to increase

due to the treatments, even when the population normality can not be assumed.

Some researchers (Blair & Sawilowsky, 1993a, 1993b, 1994) argued that the test statistic proposed by Brownie et al. (1990) is not robust with respect to Type I error when the population normality is not attainable. Simulation studies (Blair & Sawilowsky, 1993a, 1993b) showed that the Fmod test tends to yield higher Type I error rate when the population distribution deviates from normal. Blair and Sawilowsky (1994) then proposed modifications to the Fmod test so that the test would offer powerful and robust results even when the population distribution is not normal. One of the two modifications, the pFmod test, is of interest in the present paper. Testing the same null and alternative hypotheses as described above for the Fmod test, the Fmod value would be compared to a permuted referent distribution to obtain its p-value instead of using the F distribution.

$pFmod = [\Sigma n_i(x_i - x..)^2/(k-1)]/S^2_c$ -> permuted distribution

The pFmod was shown to be more powerful and robust with respect to Type I error than the Fmod test when population distribution is not normal (Blair & Sawilowsky, 1994).

Given the form of the test statistic, both the Fmod test proposed by Brownie et al. (1990) and the pFmod test proposed by Blair and Sawilowsky (1994) should yield high rejection rates when the population means or the variances are different. Therefore, when obtaining a significant result from the above two tests it is ambiguous whether the significant result is due to

the mean difference between the groups, the increased variabilities in the experimental groups, or both. Wisenbaker and Tam (1995) proposed two tests pFmod(mm) and pFmod(vv) to test the mean equality and variance equality separately. The pFmod(mm) and pFmod(vv) also use permutation to estimate the referent distribution.

$$\text{Ho: } \mu_c = \mu_{exp} \quad \text{vs. Ha: } \mu_c \neq \mu_{exp}$$

$$\text{pFmod(mm)} = \Sigma\ (x_i - x..)^2 \text{ -> permuted distribution}$$

$$\text{Ho: } \sigma^2_c \geq \sigma^2_{exp} \text{ vs. Ha: } \sigma^2_c < \sigma^2_{exp}$$

$$\text{pFmod(vv)} = \Sigma\ S^2_i/k - S^2_c \text{ -> permuted distribution}$$

By testing the null hypotheses separately, they (Wisenbaker & Tam, 1995) argued, the results would provide a clearer understanding of the situation at hand. In addition, the two tests were also robust with respect to Type I error.

This paper examined the same joint hypotheses but proposed a different approach to determining rejection (or acceptance) of the null hypotheses. The alternative hypotheses are the same as the ones Brownie et al. (1990) and Blair and Sawilowsky (1994) tested:

$$\text{Ha: } \mu_c \neq \mu_{exp} \quad \text{AND } p\sigma^2_c = \sigma^2_{exp}$$

But the null hypotheses are the negation of the alternative hypotheses. That is:

$$\text{Ho: } \mu_c = \mu_{exp} \quad \text{OR} \quad \sigma^2_c \geq \sigma^2_{exp}$$

There are three ways to satisfy the null hypotheses: (a) means are equal and variances of the experimental groups are smaller than or equal to that of the control group, or (b) means are

unequal but the variances of the experimental groups are smaller than or equal to that of the control group, or (c) means are equal but the variances of the experimental groups are greater than that of the control group. The proposed conjoint test (conjoint) takes the maximum p-value of pFmod(mm) and pFmod(vv) as its p-value. This conjoint test should provide a relatively conservative approach to testing the conjoint hypotheses but would have control on the Type I error rate when the null hypotheses were partially true.

This study investigated the robustness with respect to Type I error and power of the tests described above. The methods and results are presented below.

Method

The Statistical Analysis System (SAS) Version 6.08 procedure PROC IML was used in simulating the data. The pseudo-number generators in the SAS language NORMAL, TINV, CINV, and RANEXP were used to generate random numbers from the normal, student t, chi-squared, and exponential distributions, respectively. The programs were run on the Time Sharing Option (TSO) mainframe system.

Data were simulated for the combinations of conditions: (a) the number of groups: k=2, 4, 6, 8, and 10, (b) the number of subjects per group: n=10, 20, 40, and 80, and (c) the distribution shapes: normal, t with 3 df, chi-squared with 1 df, and exponential. Each of the 5 x 4 x 4 conditions were simulated for (a) equal mean and equal variance, (b) equal mean and unequal

variance, (c) unequal mean and equal variance, and (d) unequal mean and unequal variance. The scale difference (unequal variance) was created so that the variances of the experimental groups were twice that of the control group. The mean differences (unequal mean) were created so that the largest difference between the control and the experimental is 1σ with equal distances between the experimental groups. Nominal levels were set at .05 and .01.

Two hundred replications were generated using SAS IML and the pseudo-number generators for each of the combination of conditions as described above. Fmod, pFmod, pFmod(mm), pFmod(vv) were calculated on each replication of the generated sample data. For reference purposes, regular F statistic was also computed. P-values for the regular F and Fmod were obtained by comparing the calculated test statistic values to their appropriate F distributions. The original sample data were then permuted 500 times yielding permuted pFmod, pFmod(mm), and pFmod(vv) values. The pFmod, pFmod(mm), and pFmod(vv) values from the original sample were then compared to their permuted counterparts. The proportions of permuted values greater than or equal to the original values were the p-values for pFmod, pFmod(mm), and pFmod(vv). The maximum p-values of the pFmod(mm) and pFmod(vv) became the p-values of the conjoint statistic. Results were written to external files for further analyses. See Appendix A for an example program.

The proportions of p-values greater than or equal to the nominal levels were calculated for the Fmod, pFmod, and conjoint tests. Mixed model ANOVAs using the proportions of p-values of the Fmod, pFmod, and conjoint statistics were conducted to test the differences among the tests in the various combinations of conditions. Nominal level of .05 was used for each mixed model ANOVA.

Results

The results regarding the robustness with respect to Type I error and power were reported below.

Type I Error Rates

The rejection rates of the Fmod test (Brownie, et al., 1990), pFmod test (Blair & Sawilowsky, 1994), and the conjoint test were compared using the mixed model ANOVA where the distribution shape, number of groups, and number of subjects per group were the between subject factors and the statistical test was the repeated measure factor. Three data conditions were considered to yield Type I error rates: (a) both means and variances were equal, (b) means were unequal and variances were equal, and (c) means were equal and variances were unequal. The Type I error rates were described for the three conditions separately below.

Means and variances were equal. The regular F test fluctuated around the nominal levels (.05 and .01) as expected. The mixed model ANOVA revealed significant complex interaction effects among the Fmod, pFmod, and conjoint tests. The mixed model ANOVA results were reported in Table 1. Because the

results at the two nominal levels were similar, only results from the .05 level were reported. A closer look at the Type I error rates found that pFmod, as the regular F test, fluctuated around the nominal levels across all conditions, whereas Fmod fluctuated around the nominal levels only when the distribution shape was normal. As the distribution shapes deviated from normal, Fmod yielded liberal results as was found in the previous studies (Blair & Sawilowsky, 1993a, 1993b, 1994; Wisenbaker & Tam, 1995). The conjoint test yielded very conservative results as extreme as rejection rate = 0.00 when the nominal level was set at .01. The Type I error rates for Fmod, pFmod, and conjoint tests were reported in Table 2. Because of the similarity in the results, the following conditions were selected to be reported: k=2, 6, 10; n=10, 20, 80; distribution = normal and exponential.

Unequal means and equal variances. The regular F test yielded high rejection rates as would be expected. The mixed model ANOVA found significant complex interaction effects (see Table 3). Both Fmod and pFmod had rejection rates higher than the nominal levels. Fmod and pFmod had similar rejection rates when the distribution shape was normal. As the distribution shapes deviated from normal, Fmod became more liberal than pFmod. The conjoint test yielded conservative rejection rates across conditions. Type I error rates were reported in Table 4.

Equal means and unequal variances. The regular F test had rejection rates fluctuating around the nominal levels as expected. Again, significant complex interaction effects were found

between Fmod, pFmod, and conjoint tests (Table 5). Both Fmod and pFmod yielded rejection rates higher than the nominal levels. In general, the rejection rates of Fmod were higher than those of pFmod. The rejection rates of Fmod was similar to those of pFmod when the distribution shape was normal. As the distribution shapes deviated from normal, Fmod had more liberal rejection rates. The conjoint test still yielded very conservative rejection rates across conditions. Type I error rates were reported in Table 6.

In sum, Brownie et al.'s (1990) Fmod yielded liberal results when the distribution shapes were not normal and when the null hypotheses were only partially true. Blair and Sawilowsky's pFmod (1994) was robust with respect to Type I error rate when the distribution shapes were not normal, but it yielded liberal results when the null hypotheses were only partially true. On the contrary, the conjoint test maintained conservative results when the null hypotheses were both and partially true.

Power

Results from the data condition where both means and variances were unequal provided information on how the conjoint test fare when both null hypotheses were false. See Table 7 for the empirical power at the nominal level .05. The Fmod and pFmod tests were not included in the discussion of power because of their liberal Type I error performance.

Several observations were obtained regarding the empirical power of the conjoint test at the nominal level of .05: (a) the

power tend to increase when the number of groups increased; (b) power tend to increase when the number of subjects per group increased; (c) the power was not "high" when the number of subjects per group was small (20 or less); and (d) the distribution shapes also seemed to influence power, with normal distribution yielding the best power.

Discussion

One of the most striking results from our investigation of the actual Type I error rates associated with the tests examined here are the problems associated with the Fmod and pFmod methods when dealing with situations where it might be said that the null hypothesis is "partially" true. In those instances, only the conjoint testing procedure we proposed maintained reasonable control over empirical Type I error rates. The Fmod test yielded rates as high as .47 while the pFmod test produced rates as high as .44 when the means were equal and the variances were not equal and 1.00 for both tests when the means were unequal and the variances were equal.

It would appear that, contrary to what has been written, they actually test the null hypothesis that the means are all equal OR the variances of the experimental conditions are less than or equal to that of the control condition. Departures from either of these conditions tend to lead to large values for those test statistics. To be fair, it may be that the authors are focused on situations where, for very compelling reasons, effects on means could occur if and only if there were effects on vari-

ances. If so, our concerns about their failure to control adequately for Type I error rates under conditions of "partially" correct null hypotheses are simply not germane. However, in the conditions where increase in the experimental group variances did not lead to changes in the group means (the means were equal and the variances were unequal), both tests still yielded undesired liberal Type I error rates.

The conjoint test, on the other hand, has offered a way to dealing with the possibility that the null hypotheses could be partially true. Should one of the two null hypotheses were true, the conjoint test maintains Type I error rates that are acceptable, though a bit on the conservative side.

Reference

Blair, R. C., & Sawilowsky, S. (1993a). Comparison of two tests useful in situations where treatment is expected to increase variability relative to controls. Statistics in Medicine, 12, 2233-2243.

Blair, R. C., & Sawilowsky, S. (1993b). A note on the operating characteristics of the modified F test. Biometrics, 49, 935-939.

Blair, R. C., & Sawilowsky, S. (April, 1994). Fixing the modified F test. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Brownie, C., Boos, D. D., & Hughes-Oliver, J. (1990). Modifying the t and ANOVA F tests when treatment is expected to increase variability relative to controls. Biometrics, 46, 259-266.

Wisenbaker, J. M. & Tam, A. Y. (April, 1995). Improving the improved modified F test: A simplification of Blair and Sawilowsky's pFmod statistic. Paper presented at the annual meeting of the American Educational Research Association, San Francisco, CA.

Table 1. Mixed model ANOVA for Fmod, pFmod, and conjoint tests at nominal level .05 when both means and variances were equal.

Source: Method				
DF	Type III SS	Mean Square	F Value	Pr > F
2	.41	.21	1042.15	.0001

Source: Method*Dist				
DF	Type III SS	Mean Square	F Value	Pr > F
6	.06	.01	49.15	.0001

Source: Method*K				
DF	Type III SS	Mean Square	F Value	Pr > F
8	.03	.00	18.71	.0001

Source: Method*N				
DF	Type III SS	Mean Square	F Value	Pr > F
6	.02	.00	14.95	.0001

Source: Method*Dist*K				
DF	Type III SS	Mean Square	F Value	Pr > F
24	.01	.00	1.82	.0277

Source: Method*Dist*N				
DF	Type III SS	Mean Square	F Value	Pr > F
18	.01	.00	2.85	.0009

Source: Method*K*N				
DF	Type III SS	Mean Square	F Value	Pr > F
24	.00	.00	0.73	.8096

Source: Error		
DF	Type III SS	Mean Square
72	.01	.00

Note: Method=test, Dist=distribution, K=number of groups, N=number of subjects per group.

Table 2. Type I error rates for Fmod, pFmod, and conjoint tests at nominal level .05 when means and variances were equal.

Normal

		Fmod	pFmod	conjoint
k=2	n=10	.055	.055	.000
	n=20	.040	.045	.000
	n=80	.035	.040	.010
k=6	n=10	.055	.050	.000
	n=20	.055	.055	.000
	n=80	.055	.065	.005
k=10	n=10	.080	.080	.005
	n=20	.055	.055	.000
	n=80	.070	.050	.005

Exponential

		Fmod	pFmod	conjoint
k=2	n=10	.060	.050	.010
	n=20	.080	.060	.005
	n=80	.010	.010	.005
k=6	n=10	.135	.065	.000
	n=20	.085	.055	.005
	n=80	.105	.075	.010
k=10	n=10	.155	.020	.000
	n=20	.155	.045	.005
	n=80	.100	.050	.000

Table 3. Mixed model ANOVA for Fmod, pFmod, and conjoint tests at nominal level .05 when means were unequal and variances were equal.

Source: Method				
DF	Type III SS	Mean Square	F Value	Pr > F
2	32.09	16.04	51449.01	.0001

Source: Method*Dist				
DF	Type III SS	Mean Square	F Value	Pr > F
6	.07	.01	38.01	.0001

Source: Method*K				
DF	Type III SS	Mean Square	F Value	Pr > F
8	.15	.02	60.34	.0001

Source: Method*N				
DF	Type III SS	Mean Square	F Value	Pr > F
6	2.36	.39	1259.71	.0001

Source: Method*Dist*K				
DF	Type III SS	Mean Square	F Value	Pr > F
24	.02	.00	2.23	.0049

Source: Method*Dist*N				
DF	Type III SS	Mean Square	F Value	Pr > F
18	.07	.00	12.35	.0001

Source: Method*K*N				
DF	Type III SS	Mean Square	F Value	Pr > F
24	.09	.00	11.88	.0001

Source: Error		
DF	Type III SS	Mean Square
72	.02	.00

Note: Method=test, Dist=distribution, K=number of groups, N=number of subjects per group.

Table 4. Type I error rates for Fmod, pFmod, and conjoint tests at nominal level .05 when means were unequal and variances were equal.

Normal

		Fmod	pFmod	conjoint
k=2	n=10	.490	.485	.005
	n=20	.855	.855	.015
	n=80	1.00	1.00	.015
k=6	n=10	.340	.325	.000
	n=20	.705	.690	.015
	n=80	1.00	1.00	.025
k=10	n=10	.290	.320	.030
	n=20	.690	.705	.015
	n=80	1.00	1.00	.050

Exponential

		Fmod	pFmod	conjoint
k=2	n=10	.570	.550	.000
	n=20	.755	.725	.000
	n=80	1.00	1.00	.020
k=6	n=10	.495	.360	.010
	n=20	.650	.535	.005
	n=80	1.00	1.00	.025
k=10	n=10	.470	.245	.005
	n=20	.670	.525	.015
	n=80	1.00	1.00	.035

Table 5. Mixed model ANOVA for Fmod, pFmod, and conjoint tests at nominal level .05 when means were equal and variances were unequal.

Source: Method				
DF	Type III SS	Mean Square	F Value	Pr > F
2	2.87	1.43	3969.47	.0001

Source: Method*Dist				
DF	Type III SS	Mean Square	F Value	Pr > F
6	.15	.03	70.68	.0001

Source: Method*K				
DF	Type III SS	Mean Square	F Value	Pr > F
8	.37	.05	129.49	.0001

Source: Method*N				
DF	Type III SS	Mean Square	F Value	Pr > F
6	.03	.00	13.62	.0001

Source: Method*Dist*K				
DF	Type III SS	Mean Square	F Value	Pr > F
24	.04	.00	4.38	.0001

Source: Method*Dist*N				
DF	Type III SS	Mean Square	F Value	Pr > F
18	.01	.00	1.75	.0494

Source: Method*K*N				
DF	Type III SS	Mean Square	F Value	Pr > F
24	.02	.00	2.85	.0003

Source: Error		
DF	Type III SS	Mean Square
72	.03	.00

Note: Method=test, Dist=distribution, K=number of groups, N=number of subjects per group.

Table 6. Type I error rates for Fmod, pFmod, and conjoint tests at nominal level .05 when means were equal and variances were unequal.

Normal

		Fmod	pFmod	conjoint
k=2	n=10	.105	.100	.005
	n=20	.115	.110	.015
	n=80	.140	.135	.080
k=6	n=10	.240	.230	.005
	n=20	.275	.265	.020
	n=80	.285	.280	.055
k=10	n=10	.260	.260	.020
	n=20	.325	.325	.030
	n=80	.445	.440	.065

Exponential

		Fmod	pFmod	conjoint
k=2	n=10	.120	.075	.010
	n=20	.135	.095	.025
	n=80	.105	.090	.010
k=6	n=10	.265	.105	.000
	n=20	.265	.120	.005
	n=80	.390	.295	.055
k=10	n=10	.335	.105	.000
	n=20	.380	.225	.015
	n=80	.410	.295	.025

Table 7. The empirical power at nominal level .05.

		Normal	$\chi^2(1)$	exponential	t(3)
k=2	n=10	.04	.06	.00	.00
	n=20	.16	.10	.01	.02
	n=40	.50	.17	.03	.12
	n=80	.82	.39	.18	.32
k=4	n=10	.06	.03	.02	.02
	n=20	.28	.21	.11	.10
	n=40	.68	.37	.18	.26
	n=80	.94	.61	.35	.62
k=6	n=10	.07	.07	.02	.03
	n=20	.40	.13	.09	.17
	n=40	.74	.36	.26	.30
	n=80	.98	.63	.40	.62
k=8	n=10	.08	.06	.03	.04
	n=20	.39	.21	.14	.16
	n=40	.81	.44	.29	.40
	n=80	.97	.59	.44	.59
k=10	n=10	.09	.07	.01	.04
	n=20	.42	.26	.14	.16
	n=40	.79	.46	.23	.42
	n=80	1.00	.60	.49	.67

Appendix A

Example program: program created four groups, ten subjects per group, distributed as chi-squared with 1 df, with unequal group means, and the variances of the experimental groups are twice of that of the control group.

```
************************
** JCL for TSO system **
************************
//CHIB JOB USER=WISENBA,NOTIFY=WISENBA,MSGCLASS=6,TIME=5
 //*MAIN LINES=40
//OUT1 OUTPUT DEST=SSS03,COPIES=1
// EXEC SAS6,REGION=4000K
//SASLOG DD SYSOUT=*
//SASLIST DD SYSOUT=6,OUTPUT=*.OUT1
//TEMP DD DSN=&&TEMP,SPACE=(CYL,(30,20)),DISP=(NEW,DELETE),
// UNIT=SYSDA,VOL=SER=UGAK0C
//O2 DD DSN=WISENBA.SIMDA(CB10X4D),DISP=SHR
//SYSIN DD *

*******************************
** SAS IML to generate data **
*******************************
PROC IML;
*** Creating output file ***
CREATE OUTDAT VAR{F PF F_M PBF_M CNTF_M MM CNTF_MM VV CNTF_VV
                  JOIN};
*** Generate a 10x4 matrix with 0s ***
A=J(10,4,0);
*** Matric for creating mean difference ***
MF={0 1 2 3};
MF=SQRT(2)*MF/3;
*** Create a DO loop for 200 replications ***
DO REP=1 TO 200;
*** Generate 10x4 data points that conform to χ²(1) ***
M=CINV(UNIFORM(A),1,0);
*** Create the scale difference ***
M[,1]=M[,1]/SQRT(2)+(1-1/SQRT(2));
*** Create the mean difference ***
M=M+REPEAT(MF,10);
*** Begin calculation ***
N=NROW(M);
G=NCOL(M);
NN=N*G;
DFA=G-1;
DFE=NN-G;
GV=(M[##,]-M[+,]##2/N)/(N-1);
MSE=GV[+]/G;
MSA=((M[:,]-M[:])##2*N)[+]/(G-1);
F=MSA/MSE;
```

```
PF=1-PROBF(F,DFA,DFE);  *** <-- regular F ***
DFB=N-1;
MSC=GV[,1];
F_M=MSA/MSC;
PBF_M=1-PROBF(F_M,DFA,DFB);   *** <-- Fmod ***
MM=(M[:,]-M[:])[##];
VV=GV[,:]-GV[,1];
CNTF_M=0;
CNTF_MM=0;
CNTF_VV=0;
*** Begin permutation: 500 replications ***
DO J=1 TO 500;
P=SHAPE(M[RANK(UNIFORM(A))],10,4);
PGV=(P[##,]-P[+,]##2/N)/(N-1);
PMSA=((P[:,]-P[:])##2*N)[+]/(G-1);
PF_M=PMSA/PGV[,1];
PMM=(P[:,]-P[:])[##];
PVV=PGV[,:]-PGV[,1];
IF PF_M>F_M THEN CNTF_M=CNTF_M+1;    *** <-- pFmod ***
IF PMM>MM THEN CNTF_MM=CNTF_MM+1;    *** <-- pFmod(mm) ***
IF PVV>VV THEN CNTF_VV=CNTF_VV+1;    *** <-- pFmod(vv) ***
END;
PF_MM=CNTF_MM/500;
PF_VV=CNTF_VV/500;
JOIN=MAX(PF_MM,PF_VV);   *** <-- conjoint ***
APPEND;
END;
*** End of replication ***
*** Output data ***
DATA _NULL_;
SET OUTDAT;
FILE O2;
PUT F PF F_M PBF_M CNTF_M MM CNTF_MM VV CNTF_VV JOIN;
// EXEC TSO,COND=(0,NE)
SUBMIT SIMPGM(CB20X4)
```